AICE Mathematics AS Level

Welcome to AICE mathematics. This is a rigorous and fast paced math course so it is important to make sure that you have mastered the prerequisite skills. These necessary skills have been taught in previous math courses and it is vital that you understand these in order to be successful in this course. We will not be reviewing these at the beginning of the course and so I have provided this packet as a review so you can make sure that you fully understand these topics. It is imperative that you take the time to work through these problems, check your answers, and study anything you may have forgotten, or any skill that needs more practice.

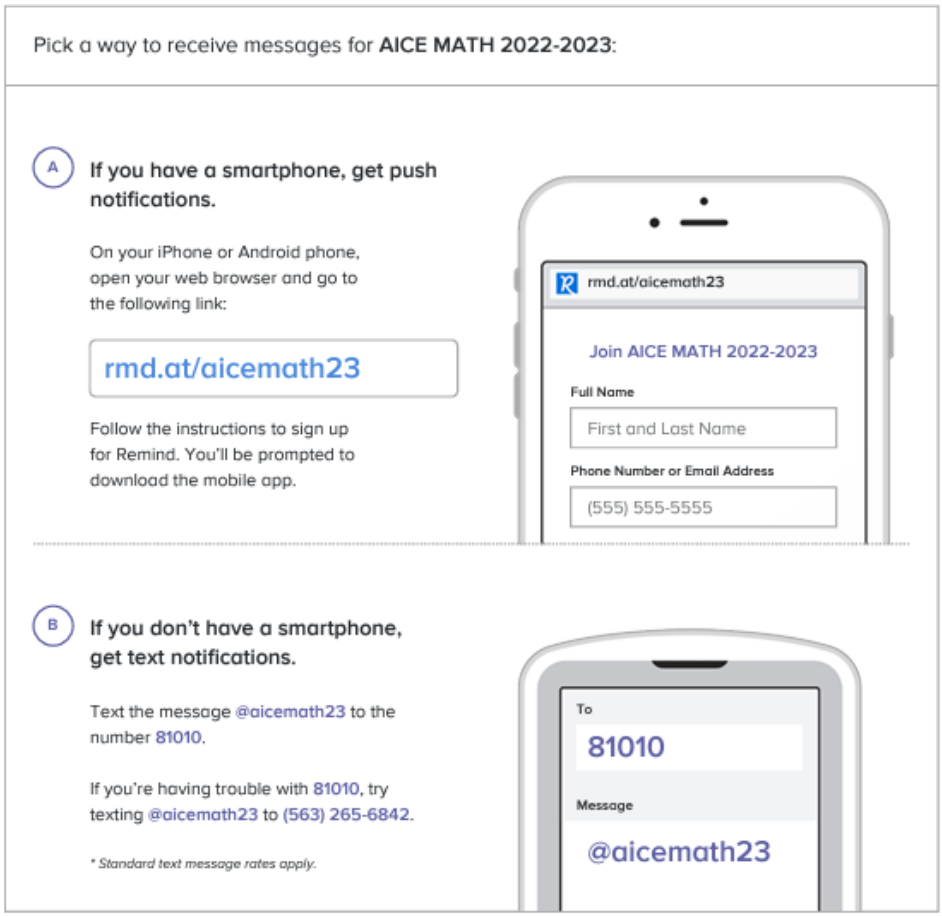
These topics will be within our first few assessments so it is important that you complete the packet and make sure you have a good understanding of the material.

I AM HERE TO HELP!!! I look forward to having you in class next year and I want you to be successful. I have provided information on where to find review videos on each topic as well as selected answer to check your work as you go. If you need further assistance or have any questions regarding the course or this packet, please don't hesitate to contact me. You may do this through remind.com (See instructions below) or by email at Benjamin.miller@ocps.net

SIGN UP FOR REMIND MESSAGES

Remind is a service that I am sure your teachers have used before, it lets me send you reminders about your work and you can also ask me questions that I can respond to. It would be very beneficial for you to go ahead and sign up in case I send anything out over the summer. Feel free to contact me through this service over the summer if you have any questions (Please know that responses won't be immediate as I won't always see them right away).

To sign up:



Calculator Information

For the exam at the end of this course you will be allowed to use a scientific calculator. Therefore, we will use one each day in the class so that you are very familiar with all of the functions on your calculator and can use it efficiently. You may use one on this material as well. **However, you will still need to show each step of the process to receive credit.**

I suggest you get your own scientific calculator to use in class each day and at home each night. I highly recommend getting the TI-36X Pro because it is one of the easiest to use with the most functionality. TI-30XS is another good one but doesn't do as much. If you don't have your own, you will be using the TI-34multiview in class which again, is ok but not the best for the exam. The TI36X pro is pictured below and is available through Walmart, Target, Amazon, and most other major stores that sell calculators, It should cost you less than $20.

https://www.amazon.com/Texas-Instruments-Engineering-Scientific-Calculator/dp/B004NBZB2Y/ref=sr_1_2?dchild=1&keywords=ti+36x+pro&qid=1621356547&sr=8-2

https://www.walmart.com/ip/Texas-Instruments-TI-36X-Pro-Scientific-Calculator/20564365

Additional Supplies:

- 1-inch Binder – You get guided notes to put in binder, keep organized. Assessments can be saved in Binder Pockets
- Tabs to keep Binders Organized
- Notebook Paper (college or wide for HW)
- Blue/Black Ink Pens
- Pencils

Videos for Review:

The site that I prefer to use for review videos and topics is a PURE Math website that is based on the content of this course. I have linked the Main Page of the website below and have also linked the main topics you will find in your summer assignment, feel free to browse the website for more videos on things you need to review.

Main Website: https://www.examsolutions.net/pure-maths-help/

- You can use the search feature to find different topics

Topics: These are linked to each topic. On the left, there will be other links with related topics as well.

1) Lines

- Gradients (slopes) of lines and parallel and perpendicular lines: https://www.examsolutions.net/tutorials/line-segment/?level=Pure&module=core&topic=1233
- Equations, midpoints, distance, etc: https://www.examsolutions.net/tutorials/equation-of-a-straight-line/?level=Pure&module=core&topic=1239

2) Solving Simultaneously (Solving Systems)

- Elimination: https://www.examsolutions.net/tutorials/elimination-method-for-linear-equations/?level=Pure&module=core&topic=1208
- Substitution: https://www.examsolutions.net/tutorials/substitution-method-for-linear-and-non-linear-equations/?level=Pure&module=core&topic=1208

3) Quadratics

- Factoring: https://www.examsolutions.net/tutorials/factorising-quadratic-expressions/?level=Pure&module=core&topic=1172
- Solving by Factoring: https://www.examsolutions.net/tutorials/solve-by-factorising/?level=Pure&module=core&topic=1186
- Completing the Square: https://www.examsolutions.net/tutorials/completing-the-square/?level=Pure&module=core&topic=1182

4) Radicals (Surds) and Exponents (Indices) –

- Surds Review: https://studywell.com/as-maths/algebra/surds/
    - https://www.examsolutions.net/tutorials/introduction-to-indices-exponents/?level=Pure&module=core&topic=1137
- Indices Review: https://studywell.com/as-maths/algebra/indices/
    - https://www.examsolutions.net/tutorials/surds-introduction-simplifying/?level=Pure&module=core&topic=14312

MUST KNOW!! (formulas must be memorized)

Slope (which will be called gradient in this course)

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Ex: find the slope of the line through the points (1, -5) and (-3, 7)

$$m=\frac{7-(-5)}{-3-1}=\frac{12}{-4}=-3$$

Distance (also used to find length)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Ex: Find the distance between (1, -5) and (-3, 7)

$$d=\sqrt{((-3)-1)^2+(7-(-5))^2}$$
$$\sqrt{(-4)^2+(12)^2}$$
$$\sqrt{160}$$
$$4\sqrt{10}$$

Midpoint

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Ex: find the midpoint of the segment with endpoints (1, -5) and (-3, 7)

$$\left(\frac{1+(-3)}{2},\frac{(-5)+7}{2}\right)$$

$\left(\frac{-2}{2},\frac{2}{2}\right)$ = (-1, 1)

Parallel Lines

- Have EXACTLY the same gradient (slope)

Ex: if line A has a gradient of $\frac{2}{3}$ and is parallel to line 2, what is the gradient of line 2?

It must be $\frac{2}{3}$

Perpendicular Lines

- Have opposite reciprocal gradients. (This means you must flip the fraction and change the sign to its opposite)
- If two lines are perpendicular, then the product of their gradients will be -1

$$m_1 \bullet m_2 = -1$$

Ex: if line A has a gradient of $-\frac{3}{4}$ and is perpendicular to line B, what is the gradient of line B?

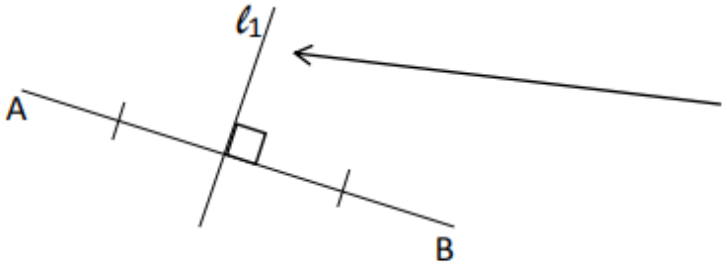
B must have a slope of $\frac{4}{3}$. (fraction is flipped, sign is opposite)

✓ Check $m_1 \bullet m_2 = -1$ → $-\frac{3}{4}\bullet\frac{4}{3}=-\frac{12}{12}=-1$

Perpendicular Bisector

A perpendicular bisector both bisects a segment and is perpendicular to it

- Bisect (goes through the midpoint)
- Perpendicular (has opposite reciprocal slopes)



Line ℓ_1 is a perpendicular bisector through segment AB

Equations of Lines

- *Point-slope form* (this is the form we will use most)
  $y-y_1=m(x-x_1)$ which passes through the point (x_1, y_1) and has gradient m

- *Horizontal lines* (slopes are 0) ⟷
  Examples: y = 2, y = – 3

- *Vertical lines* (slopes are undefined) ↕
  Examples: x = 4, x = – 7

You should be able to take point-slope form from above and write it in either of the two forms below.

- *Slope-Intercept form* $y=mx+b$, where m is gradient and b is the y-intercept
- *Standard form* $ax+by+c=0$, where a, b, and c are non-fraction constants and a is positive

Intercepts

- *x – intercepts*
  - the coordinate where the function crosses the x axis
  - to find the x – intercept, plug 0 into y and solve for x
  - written as (0, x-value) ex: (0, – 4)
- *y – intercepts*
  - the coordinate where the function crosses the y axis
  - to find the y – intercept, plug 0 into x and solve for y
  - written as (0, y-value) ex: (5, 0)

Systems of Equations

This is a very important topic that you will use throughout the entire course.

- *Points of intersection* – to find points of intersection between two lines, or between a line and a curve, get one variable in one equation by itself and then substitute (plug in) this value into the other equation

Ex: Find the coordinates of the point of intersection of the lines 2x – y = 4 and 3x + 2y = -1.

You should notice that solving for y in the first equation is easiest. When you do this you get y = 2x – 4. Take this value, 2x – 4, and plug it into y in the second equation. You would then have 3x + 2(2x – 4) = -1. Solve for x. You should find that x = 1 and y = -2. So the coordinates of the point of intersection is (1, -2)

Factor by Grouping

Example:

$6x^2+15x-21$

$=3\left(2x^2+5x-7\right)$ 1. Factor out Greatest Common Factor

$2\times-7=-14$ $+5$

Find two numbers when multiplied get -14 and when added get +5

$+7\times-2=-14$

$+7+-2=+5$

Rewrite $5x$ as $-2x+7x$

$=3\left(2x^2-2x+7x-7\right)$ 2. Split the middle term into two terms.

$=3\left[2x(x-1)+7(x-1)\right]$ 3. Rewrite the pairs of terms and take out the common factor.

$=3(x-1)(2x+7)$

Example 1: Solve

$x^2+4x-5=0$

Step 1: Write the equation in standard form:	$x^2+4x-5=0$
Step 2: Factor completely.	$(x+5)(x-1)=0$
Step 3: Apply the Zero Product Rule	$x+5=0$ *or* $x-1=0$
Step 4: Solve the linear equations in step 3.	$x=-5$ *or* $x=1$

Example 2: Solve

$3x^2+13x=10$

Step 1: Write the equation in standard form:	$3x^2+13x-10=0$
Step 2: Factor completely.	$(x+5)(3x-2)=0$
Step 3: Apply the Zero Product Rule	$x+5=0$ *or* $3x-2=0$
Step 4: Solve the linear equations in step 3.	$x=-5$ $3x-2=0$ $3x=2$ $x=\frac{2}{3}$

Rules of Indices

For $a \neq 0, b \neq 0$

Rule	Example
$a^x \times a^y = a^{x+y}$	$a^3 \times a^2 = a^{3+2} = a^5$
$a^x \div a^y = a^{x-y}$	$a^6 \div a^2 = a^{6-2} = a^4$
$(a^x)^y = a^{xy}$	$(a^2)^3 = a^{2\times 3} = a^6$
$a^0 = 1$	$a^0 = 1$
$a^{-x} = \frac{1}{a^x}$	$a^{-5} = \frac{1}{a^5}$
$a^{\frac{x}{y}} = \sqrt[y]{a^x} = (\sqrt[y]{a})^x$	$a^{\frac{3}{5}} = \sqrt[5]{a^3} = (\sqrt[5]{a})^3$

$$\frac{5x^2y^9}{15y^9x^4} = \frac{5}{15} \cdot \frac{x^2}{x^4} \cdot \frac{y^9}{y^9}$$
$$= \frac{1}{3} \cdot x^{2-4} \cdot y^{9-9}$$
$$= \frac{1}{3} \cdot x^{-2} \cdot y^0$$
$$= \frac{1}{3} \cdot \frac{1}{x^2} \cdot 1$$
$$= \frac{1}{3x^2}$$

$$(-32y^{10})^{1/5} = \sqrt[5]{-32y^{10}}$$
$$= \sqrt[5]{(-2)^5 (y^2)^5}$$
$$= -2y^2$$

1) $\sqrt{a}\sqrt{a} = a$

2) $a\sqrt{b} \times a\sqrt{b} = a^2 \times b$

3) $\sqrt{a \times b} = \sqrt{a}\sqrt{b}$

4) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

5) $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

6) $a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$

7) $\frac{a}{b + c\sqrt{n}}$ rationalize denominator by multiplying by $\frac{b - c\sqrt{n}}{b - c\sqrt{n}}$

$\frac{a}{b - c\sqrt{n}}$ rationalize denominator by multiplying by $\frac{b + c\sqrt{n}}{b + c\sqrt{n}}$

Simplify each expression: Simplify each radical first and then combine.

$$2\sqrt{50} - 3\sqrt{32} = 2\sqrt{25*2} - 3\sqrt{16*2} =$$
$$2*5\sqrt{2} - 3*4\sqrt{2} =$$
$$10\sqrt{2} - 12\sqrt{2} =$$
$$-2\sqrt{2}$$

$$\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{4}}$$
$$= \frac{5\sqrt{2}}{2}$$

Summer Assignment

Complete the following sections on your own paper. Make sure to show all of your work and keep it organized. Circle/Box up your final answers. You will hand this in when you return from summer break

Section 1: Equations of Lines

For questions 1 – 4, complete parts a – c for the line described:

a) Write the equation of the line in Point-Slope Form: $y - y_1 = m(x - x_1)$

b) Re-write it into slope-intercept form: y = mx + b

c) Re-Write it into Standard form: ax + by + c = 0 (all integer coefficients)

1) Passes through the points (-1, -1) and (4, 3)

2) Passes through (5,2) and is parallel to the line $y = \frac{6}{5}x + 2$

3) Passes through (-5, -4) and is perpendicular to $y = -\frac{4}{3}x$

4) Passes through (-1, 4) and is perpendicular to the line 2x + 3y = 8

5) Line A passes through the points (-4, -2) and (6, -7).

a) Find the gradient of line A.

b) Write the equation of line A in point-slope form.

c) Write the equation of line A in slope-intercept form.

d) Write the equation of a line that is parallel to line A.

e) Find the distance between the two points.

f) Find the midpoint of the two points.

6) The Line C is a perpendicular bisector of segment AB. The gradient of segment AB is -3 and point A has coordinates (4,7). The point (2, -1) lies on the line C.

a) What is the gradient of line C?

b) Write the equation of line C.

c) Write the equation of the line through points A and B.

d) Find the coordinates of the point where line C intersects segment AB.

e) Find the length of segment AB.

This problem may be challenging!

Drawing a picture and considering what you are given, and what you need, might help.

7) Find the equation of the line that passes through the points (5, -2) and (-4, 4), giving your answer in the form ax + by + c = 0. Hence, find the coordinates where the line intersects the x-axis (found by letting y = 0)

8) The coordinates of the line segment AB are (4, 5) and (6, -4). Find the equation of the perpendicular bisector of AB giving your answer in the form ax + by + c = 0.

Sections 2: Systems of Equations

Solve each of the following systems using substitution or elimination. Substitution is typically what we will use in the course, so practice that at least a couple of times.

9) $\begin{cases} -6x + 2y = 8 \\ 4x - y = -7 \end{cases}$

10) $\begin{cases} 4x - 2y = -12 \\ -7x - 4y = -24 \end{cases}$

11) $\begin{cases} 2y = 7x \\ 3x - 2y = 1 \end{cases}$

12) $\begin{cases} y = 2x + 3 \\ 4x - 2y = -6 \end{cases}$

13) Find the coordinates of the point where the line 2x + 3y = 7 intersects the line 4x – y = 12.

Section 3: Factoring to solve Polynomials

In this section, use the different factoring techniques to solve the given equation.

14) $x^2 - 11x + 18 = 0$

15) $3x^2 - 5x - 2 = 0$

16) $9x^2 - 3x - 30 = 0$

17) $6x^3 - 26x^2 - 20x = 0$ 18) $9x^2 - 25 = 0$ 19) $16x^3 - 64x = 0$

Section 4: Quadratic Functions in Vertex Form (Completing the Square)

Write each of the following in Vertex form (Completed Square Form): $y = a(x - h)^2 + k$, then state the following:

a) The Vertex

b) The Axis of Symmetry

c) The range of the function

20) $x^2 + 6x + 8 = 0$ 21) $x^2 - 2x - 15 = 0$ 22) $-x^2 + 2x + 63 = 0$

23) $5x^2 - 6x - 11 = 0$ 24) $6x^2 + 9x - 27 = 0$ 25) $-4x^2 + 10x - 15 = 0$

Section 5: Radical and Exponential Expressions

In each question, try to simplify the given radical or exponential expression as much as possible. Answers should only contain positive exponents.

26) $(-4x^2)(-2x^{-2})$ 27) $\left(\frac{2w^{-3}}{m^4}\right)^{-5}$ 28) $\frac{(12m^2n^6)^2}{8m^4n^7}$

29) $(-5x^6)(2x^{-4}y)(-6x^{-3}y^4)$ 30) $\sqrt[3]{40x^8y^{12}}$ 31) $\sqrt[3]{25xy^8} \cdot \sqrt[3]{5x^4y^3}$

32) $\frac{\sqrt[3]{192x^8}}{\sqrt[3]{3x}}$ 33) $(3+2\sqrt{5})(2-4\sqrt{5})$

For questions 34 – 36, Rationalize the Denominator, final answers should not have radicals in the denominator.

34) $\frac{\sqrt{3}}{\sqrt{5}}$ 35) $\frac{\sqrt{x^3}}{\sqrt{5xy}}$ 36) $\sqrt{\frac{9x}{2}}$

Selected Answers:

This section has some of the answers to the summer assignment but not all. This is so you can make sure you are doing things correctly and if you have questions, please reach out.

****For answers given in point-slope form, the first point was used to create it, as long as your slope intercept and standard forms match you should be good.**

1. a) $y+1=-\frac{2}{5}(x+1)$ b) $y=-\frac{2}{5}x-\frac{7}{5}$ c) $2x+5y+7=0$

4. a) $y-4=\frac{3}{2}(x+1)$ b) $y=\frac{3}{2}x+\frac{11}{2}$ c) $3x-2y+11=0$

7. Standard form: 2x + 3y – 4 = 0, Value of x-intercept (plug in y = 0): x = 2

10. (0, 6)

12. infinite Solutions (same line)

14) Factors: (x – 9)(x – 2), solutions are x = 9, and x = 2

16) Factors: 3(3x + 5)(x – 2), solutions are x = -5/3 and x = 2

19) Factors: $16x^2(x + 2)(x – 2)$, solutions are x = 0, x = -2, and x = 2

22) Vertex Form: $-(x – 1)^2 + 64$, Vertex: (1, 64), Axis of Symmetry: x = 1, range: $y \leq 64$

24) Vertex Form: $6(x + \frac{3}{4})^2 - \frac{243}{8}$, Vertex: $(\frac{-3}{4}, \frac{-243}{8})$, Axis of Symmetry: $x = \frac{-3}{4}$, range: $y \geq \frac{-243}{8}$

27) $\frac{\mathbf{w^{15}m^{20}}}{\mathbf{32}}$ **29)** $\frac{\mathbf{60y^5}}{\mathbf{x}}$

31) $5xy^3\sqrt[3]{x^2y^2}$ **33)** $-34 - 8\sqrt{5}$

35) $\frac{\mathbf{x}\sqrt{\mathbf{5y}}}{\mathbf{5y}}$